GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 4260

Problem Set #12

Date assigned: April 19, 2017
Date due: Not collected, but covered on final

Reading: Read Sections 10.4, 10.5, 10.6 in Stark and Woods.

Reminder: Final Exam: Wednesday, April 29, 2016 at 2:50 pm. You may bring three 8 1/2" by 11" handwritten sheets with you. Tables 8.4-1, 9.5-1, 9.5-2 will be provided as well as a table of pdf's and properties, ACF-PSD pairs, and trigonometric identities. Check the course website for a replica.

Problem 12.1:

Work problem 10.7 in Stark and Woods

Problem 12.2:

Work problem 10.22 in Stark and Woods

Problem 12.3:

Work problem 10.26 in Stark and Woods

Problem 12.4:

Work problem 10.27 in Stark and Woods

Problem 12.5:

Work problem 10.44 in Stark and Woods

Problem 12.6:

Find the eigenfunctions and eigenvalues of the Karhunen-Loève expansion for:

(a) white noise.

(b) a process where $R_{XX}(t_1, t_2) = f(t_1)f(t_2)$.

Problem 12.7:

Consider a random process $X(t)$ with a 4-term Karhunen-Loève expansion over $[0, T]$:

$$X(t) = \sum_{i=1}^{4} X_i \phi_i(t) , \ 0 \le t \le T$$

The X_i are zero-mean, with $E[X_i X_j] = \lambda_i \delta_{ij}$. Also

$$\int_0^T \phi_i(t)\phi_j^*(t) = \delta_{ij}.$$

(a) Express the expected energy in $X(t)$ over $[0, T]$ in terms of the λ_i's.

(b) Suppose $X(t)$ is input to a linear system with output $Y(t)$. Specifically:

$$Y(t) = \int_0^T h(t, u)X(u)du.$$

Let $h(t, u) = \phi_1(t)\phi_3^*(u) + \phi_2(t)\phi_3^*(u)$. Express $Y(t)$ in terms of λ_i and $\phi_i(t)$ $i = 1, 2, 3, 4$.

Problem 12.8:

Work problem 10.4 in Stark and Woods